

**University of Mumbai**  
**Examinations Summer 2022**

Time: 2 hour 30 minutes

Max. Marks: 80

Q1.	<b>Choose the correct option for following questions. All the Questions are compulsory and carry equal marks</b>	
1.	$L\{t^3 + e^{-3t}\}$ equals	
Option A:	$\frac{6}{s^4} + \frac{1}{s+3}$	
Option B:	$\frac{6}{s^4} - \frac{1}{s-3}$	
Option C:	$\frac{4!}{s^4} + \frac{1}{s+3}$	
Option D:	$\frac{4!}{s^4} + \frac{1}{s-3}$	
2.	$L\{e^{2t} \cos 3t\}$ equals	
Option A:	$\frac{s+2}{(s+2)^2+9}$	
Option B:	$\frac{3}{(s+2)^2+9}$	
Option C:	$\frac{s-2}{(s-2)^2+9}$	
Option D:	$\frac{3}{(s-2)^2+9}$	
3.	$L^{-1}\left\{\frac{2}{(s-2)(s+3)}\right\}$ equals	
Option A:	$\frac{2}{5}(e^{-2t} - e^{3t})$	
Option B:	$\frac{2}{5}(e^{2t} - e^{-3t})$	
Option C:	$2(e^{2t} - e^{-3t})$	
Option D:	$\frac{2}{5}(e^{2t} + e^{-3t})$	
4.	$L^{-1}\left\{\frac{s-1}{s^2-2s+5}\right\}$ equals	
Option A:	$e^{-t} \cos 2t$	
Option B:	$e^t \sin 2t$	
Option C:	$e^{-t} \sin 2t$	
Option D:	$e^t \cos 2t$	
5.	If $u = y^2 - 2xy + ax^2 + 5x - 3y$ is harmonic, then	
Option A:	$a = 1$	
Option B:	$a = -1$	
Option C:	$a = -2$	
Option D:	$a = 2$	
6.	The function $f(z) = \frac{z}{(z-3)^2(z+i)^3}$ has	
Option A:	Poles of order 3 and 2 respectively at $z = -i$ and $z = 3$	

Option B:	Poles of order 2 and 3 respectively at $z = -3$ and $z = i$
Option C:	Poles of order 3 and 2 respectively at $z = i$ and $z = -3$
Option D:	Poles of order 2 and 3 respectively at $z = 3$ and $z = i$
7.	Suppose $f(x) = x$ in $(0,2)$ . Then the Fourier coefficient $a_0$ , where $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x + \sum_{n=1}^{\infty} b_n \sin n\pi x$ is the Fourier Series of $f(x)$ is equal to
Option A:	0
Option B:	1
Option C:	2
Option D:	-1
8.	The functions $f(x) = 1$ and $g(x) = x$ are defined in the interval $(-1,1)$ . Then
Option A:	$f(x)$ and $g(x)$ are orthonormal in $(-1,1)$
Option B:	$f(x)$ and $g(x)$ are orthogonal, but not orthonormal in $(-1,1)$
Option C:	$f(x)$ and $g(x)$ are not orthogonal in $(-1,1)$
Option D:	$f(x)$ and $g(x)$ are orthonormal, but not orthogonal in $(-1,1)$
9.	Identify the one- dimensional wave equation among the following:
Option A:	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
Option B:	$5 \frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2}$
Option C:	$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 y}{\partial x^2}$
Option D:	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
10.	Suppose the two regression coefficients are $b_{yx} = \frac{-1}{2}$ , $b_{xy} = \frac{-1}{4}$ then the correlation coefficient $r$ is
Option A:	$-\sqrt{\frac{1}{8}}$
Option B:	$\pm \sqrt{\frac{1}{8}}$
Option C:	$\sqrt{\frac{1}{8}}$
Option D:	$\frac{1}{8}$

## Subjective/Descriptive questions

<b>Q2</b> (20 Marks)	<b>Solve any Four out of Six (5 marks each)</b>
A	Evaluate using Laplace Transforms: $\int_0^{\infty} e^{-4t} \cos 2t \sin 5t dt$
B	Find $L^{-1}\left\{\frac{s}{(s^2-8s+25)}\right\}$
C	Find a, b, c, d if $f(z) = (x^3 + axy^2 + by^2 + x^2) + i(cx^2y - y^3 + dxy)$ is analytic.
D	Suppose $f(a) = \int_C \frac{2z^2-3z+7}{z-a} dz$ where $a$ lies inside C and C is the circle $ z - 1  = 2$ . Evaluate $f(2)$ and $f'(2)$
E	Obtain the Fourier series of $f(x) = x^3, -\pi \leq x \leq \pi$
F	Solve using Bender-Schmidt method: $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ ; subject to the conditions: $u(0, t) = 0; u(4, t) = 0; u(x, 0) = x^2(16 - x^2)$ taking $h = 1$ upto 2 seconds

<b>Q3</b> (20 Marks)	<b>Solve any Four out of Six (5 marks each)</b>														
A	Obtain $L\left\{\int_0^t u \sin 2u du\right\}$														
B	Obtain the analytic function whose real part is $e^{-x} \cos y$ .														
C	Fit a straight line to the following data by the method of least squares where Y is the dependent variable: <table border="1" style="margin: 10px auto; width: 80%;"> <thead> <tr> <th>X</th> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> </thead> <tbody> <tr> <th>Y</th> <td>12.2</td> <td>13.4</td> <td>11.8</td> <td>14.6</td> <td>14.8</td> <td>16.2</td> </tr> </tbody> </table>	X	1	2	3	4	5	6	Y	12.2	13.4	11.8	14.6	14.8	16.2
X	1	2	3	4	5	6									
Y	12.2	13.4	11.8	14.6	14.8	16.2									
D	Evaluate $\int_C \frac{z}{(z-4)(z+2)} dz$ using Cauchy's Residue Theorem where C is the circle $ z - 3  = 2$														
E	Obtain the Half Range Fourier Cosine series of $f(x) = x - x^2, 0 < x < 1$														
F	Solve using Crank-Nicolson formula: $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0, 0 \leq x \leq 1$ ; subject to the conditions: $u(0, t) = 0; u(1, t) = 100 t; u(x, 0) = 0$ taking $h = 0.25$ for one time-step														

<b>Q4</b> <b>(20 Marks)</b>	<b>Solve any Four out of Six (5 marks each)</b>					
A	Find: $L^{-1}\left\{\frac{s}{(s^2+4)(s^2+1)}\right\}$ using convolution theorem					
B	Obtain the Bilinear transformation that transforms the points $z = -1, 0, 1$ respectively to the points $w = \infty, -1, 0$					
C	Obtain the Laurent series for the function $f(z) = \frac{1}{z(z-1)}$ around $z = 0$ in the region $ z  > 1$					
D	Obtain the complex form of Fourier series of $f(x) = e^{4x}$ in $(0, 4)$					
E	Obtain the Spearman's rank correlation coefficient of the following marks in Subjects X and Y :					
	X	20	15	18	15	12
	Y	16	19	16	14	14
F	Obtain the solution of the one – dimensional heat equation: $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ using the boundary conditions: $u(0, t) = 0; u(l, t) = 0; u(x, 0) = x, 0 < x < l$ where $l$ is the length of the rod					