

Duration: 3 hours

Max. Marks: 80

N.B. (1) Question No. 1 is **COMPULSORY**.(2) Answer **ANY THREE** questions from Q.2 to Q.6.

(3) Figures to right indicate full marks.

- Que. 1**
- a. Find Laplace Transform of  $t \cos 4t \cdot \cos 7t$  5
- b. Find Fourier series expansion of  $f(x) = x$  in  $(-\pi, \pi)$  5
- Find the orthogonal trajectory of the family of curves given by 5
- c.  $2x - x^3 + 3xy^2 = a$
- d. If  $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ , Find eigen values of  $A^3 - 3A^2 + 5A$  5
- Que. 2**
- a. Obtain Fourier series expansion for  $f(x) = x^2$  in  $(0, 2\pi)$  6
- By using partial fractions, find the inverse Laplace transform of 6
- b.  $\frac{s^2}{(s^2+9)(s^2+16)}$
- Find the eigenvalues and the eigenvectors of the matrix 8
- c.  $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$
- Que. 3**
- a. Find the analytic function whose real part is  $\frac{\sin 2x}{\cosh 2y + \cos 2x}$  6
- b. Find the Laplace transform of  $\sinh^5 t$  6
- Using Bender Schmidt method, solve  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ , subject to 8
- c.  $u(0, t) = 0, u(1, t) = 0, u(x, 0) = \sin \pi x \quad 0 \leq x \leq 1$
- Que. 4**
- a. By using Laplace transform, evaluate,  $\int_0^\infty \frac{\cos 3t - \cos 5t}{t} dt$  6
- Find a, b, c, d, e if 6
- b.  $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$  is an analytic function. Obtain the half range cosine series of  $f(x) =$  8
- c.  $\begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$

**Que. 5** a. Find the analytic function  $f(z) = u + iv$ , in terms of  $z$ , if **6**

$$u = y^3 - 3x^2y$$

b. If  $L\{f(t)\} = \frac{s}{s^2 + s + 4}$ , find  $L\{e^{-2t} f(2t)\}$  **6**

c. Determine if the matrix  $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$  diagonalizable, hence **8**  
 find its diagonal matrix  $D$  and modal matrix

**Que. 6** a. Determine the Half Range Sine Series for  $f(x) = \frac{x(\pi^2 - x^2)}{12}$ , where **6**  
 $0 < x < \pi$ .

b. Find inverse Laplace transform of  $\cot^{-1}\left(\frac{s+3}{2}\right)$  **6**

c. Using Crank- Nicholson simplified formula, solve  $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ , **8**  
 $u(0, t) = 0$ ,  $u(4, t) = 0$ ,  $u(x, 0) = \frac{x}{3}(16 - x^2)$  for one step for time.

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