

(3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is Compulsory.

2) Answer any THREE questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) What is the value of  $\int_0^{1+i} (x - y + ix^2) dz$  along the line from  $z = 0$  to  $z = 1 + i$  (5)

(b) Find a and b such that  $\vec{F} = (axy + z^3)i + x^2j + bz^2xk$  is irrotational (5)

(c) A random variable X has probability mass function  $p(x) = kx^3$ ;  $x=1,2,3,4$  then find the value of k, mean, variance. (5)

(d) Find the probability that at most 4 defective bulbs will be found in a box of 200 bulbs if it is known that 2% of the bulbs are defective. (5)

Q.2 (a) Find the rank correlation coefficient between X and Y; (6)

X	17	13	15	16	6	11	14	9	7	12
Y	36	46	35	24	12	18	27	22	2	8

(b) A random variable has the MGF  $M_X(t) = \frac{3}{3-t}$ . Find mean and Variance of X. (6)

(c) Obtain Laurent's series expansions of  $f(z) = \frac{z-1}{z^2-2z-3}$ ;  $|z| > 3$ . (8)

Q.3 (a) A coin is tossed. If it turns up heads two balls are drawn from urn A otherwise two balls are drawn from urn B. Urn A contains 3 black and 5 white balls. Urn B contains 7 black and one white ball. What is the probability that urn A was used, given that both balls drawn are black? (6)

(b) Fit a straight line  $y = a + bx$  into the given data. (6)

x:	10	20	30	40	50
y:	22	23	27	28	30

(c) Prove that  $\vec{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$  is irrotational. Find scalar potential of  $\vec{F}$ . Hence find the work done of moving particle from (1,0,2) to (0,1,1). (8)

- Q.4** (a) Using Green's Theorem evaluate  $\int_c (xy + y^2)dx + x^2 dy$  and  $c$  is closed curve of the region bounded by  $y = x$  and  $y = x^2$ . (6)
- (b) A machinist is expected to make engine parts with axle diameter of 1.75 cm. A random sample of 10 parts shows a mean diameter of 1.85 cm, with a S.D of 0.1 cm. Based on this sample, would you say that the work of the machinist is inferior? (6)
- (c) A random variable  $X$  follows a normal distribution with mean 14 and standard deviation 2.5 find (1)  $P[X < 8]$  (2)  $P[X > 18]$  (3)  $P[12 < X < 15]$  Given: Area between  $z=0$  and  $z=2.4$  is 0.4918 ; Area between  $z=0$  and  $z=1.6$  is 0.4452 ; Area between  $z=0$  and  $z=0.8$  is 0.2882 ; Area between  $z=0$  and  $z=0.4$  is 0.1554. (8)

- Q.5** (a) The standard deviation from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regarded as drawn from normal population with same standard deviation? ( $F_{(8,12)}(0.025) = 3.51, F_{(12,8)}(0.025) = 4.20$ ) (6)
- (b) Use Gauss's Divergence Theorem to evaluate  $\iint_S \bar{N} \cdot \bar{F} ds$ , where  $\bar{F} = 4xi - 2y^2j + z^2k$  and  $S$  is region bounded by  $x^2 + y^2 = 4, z = 0, z = 4$ . (6)
- (c) Obtain both Line of regressions for the data given below (8)  
Given  $\sum X = 250 ; \sum Y = 300 ; \sum XY = 7900 ; \sum X^2 = 6500 ; \sum Y^2 = 10000$  and  $n = 10$  (in usual notation)

- Q.6** (a) Evaluate Value of  $\int_C \frac{\sin 2z dz}{(z + \pi/3)^4}$  where  $C: |z| = 2$  (6)
- (b) The following data find the correlation coefficient to marks obtained by 11 students in 2 tests, one held at the beginning of the year and the other at the end of the year after intensive coaching: (6)

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

- (c) A die was thrown 132 times and the following frequencies were observed. (8)

No. obtained	1	2	3	4	5	6	Total
Frequency	15	20	25	15	29	28	132

Test the hypothesis that the die is unbiased at 5% level of significance.

(Given: Table value of  $\chi^2$  at 5% level of significance and 5 degree of freedom is 11.07)

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