[3 hours] **[Total Marks 80]**

N.B. (1) Question No. 1 is compulsory
(2) Attempt any THREE from the remaining Q2 to Q6
(3) Use of statistical table is allowed.

1. (a) Prove that **F**= (x+2y+4z)i+(2x-3y-z)j+(4x-y+2z)k is solenoidal (5)

(b) Maximize $Z = x_1 - 2x_2 + 4x_3$
Subject to constraints $x_1 + 2x_2 + 2x_3 + 8x_4 = 7$; $3x_1 + 4x_2 + 6x_3 = 15$. (5)
$x_1, x_2, x_3 \geq 0$ then find (i) all basic solutions

(c) A continuous random variable with p.d.f. $f[x] = k x^2(1-x^3)$, $0 \leq x \leq 1$. Find k (5)
Find mean and variance

(d) Use Cayley - Hamilton theorem to find $2A^4 - 5A^3 - 7A + 6I$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ (5)

2. (a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (6)

(b) Using Green's theorem evaluate $\int (xy + y^2)dx + x^2 dy$ where c is the closed curve (6)
of the region bounded by $y = x$ and $y = x^2$

(c) using Dual Simplex method solve minimize $z = -3x_1 - 2x_2$ sub. $x_1+2x_2 \geq 1$, $x_1+x_2 \leq 7$, (8)
$x_1+2x_2 \leq 10$, $x_1, x_2 \geq 0$

3. (a) If the vector field $\bar{F}$ is irrotational find the constants a,b,c where
$\bar{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z) k$. (6)

(b) By using Big M method solve Minimize $Z = 2x_1 + 3x_2$ Subject to $x_1 + x_2 \geq 5$ (6)
$x_1 + 2x_2 \geq 6$; $x_1\ x_2 \geq 0$

(c) In a factory production can be achieved by four different workers on five different types of machines a sample study was made for two fold objectives of examining whether the four differ with respect to mean productivity and whether the mean productivity is the same for five different machine. The researcher involved in this study while analysing the collected data ,reports as follows, (8)

1. Sum of squares of variances between machines =35.2
2. Sum of squares of variances between workers =53.8
3. Sum of squares of variances =174.2

Construct an ANOVA table for the given information and draw the interference at 5 % level.

4. (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ then find A^{100} (6)

(b) By using simplex method solve Maximize $Z = 4x_1 + 8x_2 + 5x_3$ (6)
Subject to $x_1 + 2x_2 + 3x_3 \leq 18$; $2x_1 + 6x_2 + 4x_3 \leq 15$;
$x_1 + 4x_2 + x_3 \leq 6$; $x_1, x_2, x_3 \geq 0$

(c) Individuals are chosen at random from population and their heights are found to 63,63,64,65,66,69,69,70,71,70 inches. Discuss the suggestions that mean height of the population is 65 inches. (8)

5. (a) Show that the matrix A is derogatory and find its minimal polynomial

$$A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$ (6)

(b) It is shown that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i)at least 3 (ii) exactly 3 (iii) at most three defective items in a consignment of 1000 packets using Poisson Distribution (6)

(c) Based on the following data determine if there is a relation between literacy and smoking (8)

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

using $\chi 2$ test.

6. (a) Use Gauss divergence theorem to evaluate where $\iint \bar{N} \cdot \bar{F}\, ds$ where (6)
$\bar{F} = \left(4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}\right)$ and S is the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$

(b) Can it be concluded that average life span of an indian is more than 70 years. ,if a random sample of 100 indians has an average life span of 71.8 years with standard deviation of 7.8 years? (6)

(c) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1$ into canonical form and hence find rank, index and signature of the matrix (8)

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