University of Mumbai

Examinations Summer 2022

Time: 2hour 30 minutes Max. Marks: 80

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Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	For a matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, Eigen values are
Option A:	1, 2, 3
Option B:	-1, 3, -2
Option C:	1, -3, 2
Option D:	-6,2,3
2.	Any function which satisfies Euler's equation is called
Option A:	Lagrange's function
Option B:	Euler's function
Option C:	extremal
Option D:	Diagonalizable
3.	If $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, then e^A is
Option A:	$e^{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}}$
Option B:	$\begin{bmatrix} e & 0 \\ 0 & e^2 \end{bmatrix}$
Option C:	$\begin{bmatrix} e & 1 \\ 1 & e^2 \end{bmatrix}$
Option D:	$e\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
4.	Problems for finding extremal with constraints can be solved by using
Option A:	Rayleigh Ritz method
Option B:	Lagrange's multipliers
Option C:	Runge -Kutta method
Option D:	Cauchy's method
5.	Which is suitable formula to find Extremals $\int_{x1}^{x2} \quad (1 + x^2 y')y'dx$ is given by
Option A:	$\frac{\partial F}{\partial y'} = c$
Option B:	$\frac{\partial F}{\partial y'} - \frac{d}{dx}\left(\frac{\partial F}{\partial y}\right) = 0$
Option C:	$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$
Option D:	$\frac{\partial F}{\partial y} = c$
6.	Cauchy-Schwartz inequality is
Option A:	$\|u.v\| \leq \|\|u\|\| \ \|\|v\|\|$

Option B:	$\|u.v\| < \|\|u\|\| \ \|\|v\|\|$
Option C:	$\|u.v\| > \|\|u\|\| \ \|\|v\|\|$
Option D:	$\|u.v\| \geq \|\|u\|\| \ \|\|v\|\|$
7.	The value of $\int \bar{z}dz$, along upper half of the circle |z|=1
Option A:	$2\pi i$
Option B:	πi
Option C:	0
Option D:	$-2\pi i$
8.	Cayley -Hamilton theorem states
Option A:	The degree of minimal polynomial is always the characteristics polynomial
Option B:	Every square matrix is similar to a diagonal matrix
Option C:	For every square matrix Algebraic multiplicity is equal to Geometrical multiplicity
Option D:	Every square matrix satisfies its own characteristic equation
9.	The curve on which the function $\int_{x_1}^{x_2} \sqrt{1+y'^2}\,dx$ is extremal
Option A:	Straight line
Option B:	Parabola
Option C:	Ellipse
Option D:	Hyperbola
10.	Find value of k if U = (2 , 1 , 3) and V = (4 , 7 , k) are orthogonal.
Option A:	K = -4
Option B:	K = 4
Option C:	K = 5
Option D:	K = -5

Q2	Solve any Four out of Six 5 marks each
A	Verify Cayley-Hamilton Theorem for the matrix A and hence, find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.
B	Let $V = R^2$ and define addition and multiplication as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_1)$ $k(x_1, y_1) = (3kx_1, 3ky_1)$ Prove that it is not a vector space
C	Obtain Laurent's series expansion of $f(z) = \frac{1}{z^2+4z+3}$ $when \quad (i) 1 < \|z\| < 3 \qquad (ii) \|z\| > 3$
D	Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$
E	Find the extremal of $\int_{x_1}^{x_2} (y^2 + y'^2 + 2ye^x)\, dx$
F	Find the rankcorrelation coefficient from the following data.

X	52	63	45	36	72	65	45	25
Y	62	53	51	25	79	43	60	33

Q3.	Solve any Four out of Six 5 marks each
A	Construct an orthogonal basis of R^2 using Gram-Schmidt process to S={(3,1),(2,2)}
B	Find the possible Laurent's expansions of the function $\frac{z-1}{(z-3)(z+1)}$ about z = 0
C	Find the extremal of $\int_0^{3\pi/2} (y^2 - y'^2)\, dx$ given y(0) = 0 , y(3π/2)=1
D	Using Rayleigh -Ritz method find an approximate solution for extremal of $\int_0^1 (2xy - y^2 - y'^2)\, dx$ with y(0)=0 and y(1)=0
E	Evaluate $\int_0^{2+i} (\bar{z})^2 dz$ along the line x=2y
F	Find the Eigen Values and Eigen vectors of the following matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Q4.	Solve any Four out of Six 5 marks each
A	Find the Eigen values and Eigen vectors of the following matrix $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

<table>
<tr><td>B</td><td>Find the extremal of $\int_{x_1}^{x_2} y\sqrt{(1+y'^2)}\ dx$</td></tr>
<tr><td>C</td><td>The probability density function of a random variable is given by x. Find the mean & variance for f(x) = $\begin{cases} 0, & x \le 0 \\ kxe^{-x/3}, & x > 0 \end{cases}$.</td></tr>
<tr><td>D</td><td>Find Karl Pearson’s coefficient of correlation between X and Y.
<table>
<tr><td>X</td><td>35</td><td>38</td><td>43</td><td>30</td><td>54</td><td>68</td><td>70</td><td>92</td><td>44</td><td>56</td></tr>
<tr><td>Y</td><td>51</td><td>37</td><td>48</td><td>62</td><td>93</td><td>73</td><td>56</td><td>72</td><td>70</td><td>92</td></tr>
</table></td></tr>
<tr><td>E</td><td>Evaluate $\int_0^{3+i} z^2 dz$ along the parabola $x = 3y^2$.</td></tr>
<tr><td>F</td><td>Construct an orthonormal basis of R^3 using Gram-Schmidt process to $S = \{(1,2,0),(0,3,1)\}$.</td></tr>
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