

Duration: 3hrs

[Max Marks:80]

- N.B. : (1) Question No 1 is Compulsory.
 (2) Attempt any **three** questions out of the remaining five.
 (3) All questions carry equal marks.
 (4) Assume suitable data, if required and state it clearly.

- 1 (All questions of Q1 are compulsory) [20]
- a Obtain $L\{f(3t)\}$ if $L\{f(t)\} = \frac{s^2 - 2s + 7}{(s^3 + 5s^2 - s + 3)}$ [05]
- b Obtain the Fourier series of $f(x) = x$, $-2 \leq x \leq 2$ [05]
- c Find a, b, c, d if $f(z) = (ax^3 + bxy^2 + x) + i(3x^2y + cy^3 + dy)$ is analytic. [05]
- d For the regression lines $2x - y + 5 = 0$ and $3y - x + 7 = 0$ find [05]
 (i) the means of x and y and (ii) correlation coefficient r
- 2 a Find: $L^{-1}\left\{\frac{s}{(s^2+9)(s^2+1)}\right\}$ using convolution theorem. [06]
- b Solve using Bender-Schmidt method: $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$; subject to the conditions: [06]
 $u(0, t) = 0$; $u(3, t) = 0$; $u(x, 0) = x^2(9 - x^2)$
 taking $h = 1$ upto 2 seconds
- c Find orthogonal trajectories of $x^2 - y^2 - 2xy = c$ [08]
- 3 a Find the Complex form of Fourier series for $f(x) = e^x$ in $(0, 2\pi)$ [06]
- b Obtain the Laurent's series for [06]
 $f(z) = \frac{1}{(z+1)(z-4)}$ in $1 < |z + 1| < 3$
- c Obtain [08]
 (i) $L\{e^t \cos 2t\}$ (ii) $L\left\{\int_0^t \cos^2 2t \, dt\right\}$

4 a Find: $L^{-1}\left\{\frac{s+2}{s^2-4s+5}\right\}$ [06]

b Fit a straight line to the following data by the method of least squares where Y is the dependent variable: [06]

X	1	2	3	4	5	6
Y	11.7	14.6	13.2	15.4	16.8	15.8

c Evaluate $\int_C \frac{1}{(z-1)(z-5)} dz$ where C is the circle $|z| = 2$ [08]

5 a Find an analytic function whose real part is $\cos 2x \cosh 2y$ [06]

b Solve using Crank-Nicolson formula: [06]

$$\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0, \quad 0 \leq x \leq 1; \text{ subject to the conditions:}$$

$$u(0, t) = 0; u(1, t) = 0; u(x, 0) = 100x(1-x) \text{ taking}$$

$$h = 0.25 \text{ for one step}$$

c Obtain the Half Range Fourier Cosine Series of [08]

$$f(x) = x, \quad 0 \leq x \leq 2 \text{ and deduce that } \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$$

6 a Obtain the Spearman's rank correlation coefficient of the following marks in Subjects X and Y : [06]

X	18	14	18	12	11	11
Y	15	20	16	10	10	15

b Obtain the Bilinear transformation that transforms the points $z = 1, 2, 0$ respectively to the points $w = 1, \infty, 2$ [06]

c Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ [08]